KOMPLEKS O‘ZGARUVCHILI FUNKSIYA HOSILASI, I VA II TUR KONFORM AKSLANTIRISHLAR



Adilov Boburjon Baxridin o‘g‘li
Jizzax politexnika instituti “Oliy matematika” kafedrasi
dotsent v.v.b., p.f.f.d., (PhD)

Annotatsiya

Ushbu maqolada kompleks o‘zgaruvchili funksiya hosilasi I va II tur konform akslantirishlar misollar orqali tushuntirib berilgan.

Kalit so‘zlar: funksiya, argument, akslantirish, funksiya differensiali, hosila.

Аннотация

В этой статье производная функции комплексной переменной объясняется на примерах конформных отражений типов I и II.

Ключевые слова: функция, аргумент, отражение, дифференциал функции, производная.

Annotation

In this article, the derivative of a function of a complex variable is explained using examples of conformal maps of types I and II.

Keywords: function, argument, reflection, function differential, derivative.

Biror E kompleks sohada $w = f(z)$ funksiya berilgan bo‘lsin va bu sohaning biror z_0 nuqtadagi argument va funksiya orttirmalari quyidagicha bo‘lsin:

$\Delta z = z - z_0$, $\Delta w = f(z_0 + \Delta z) - f(z_0)$

Ta’rif. Agar Δz har qanday yo‘l bilan nolga intilganda $\frac{\Delta w}{\Delta z}$ nisbat faqat birgina aniq limitga intilsa, u limintning qiymati $f(z)$ funksiyasiyaning z_0 nuqtadagi hosilasi deyiladi va u

w' ; $f'(z_0), \frac{\Delta w}{\Delta z}, \frac{df}{dz}$ kabi belgilanib, $f'(z_0) = \lim\limits_{\Delta z \to 0} \frac{\Delta w}{\Delta z} = \lim\limits_{\Delta z \to 0} \frac{f(z_0 + \Delta z) - f(z_0)}{\Delta z}$ (1.1)

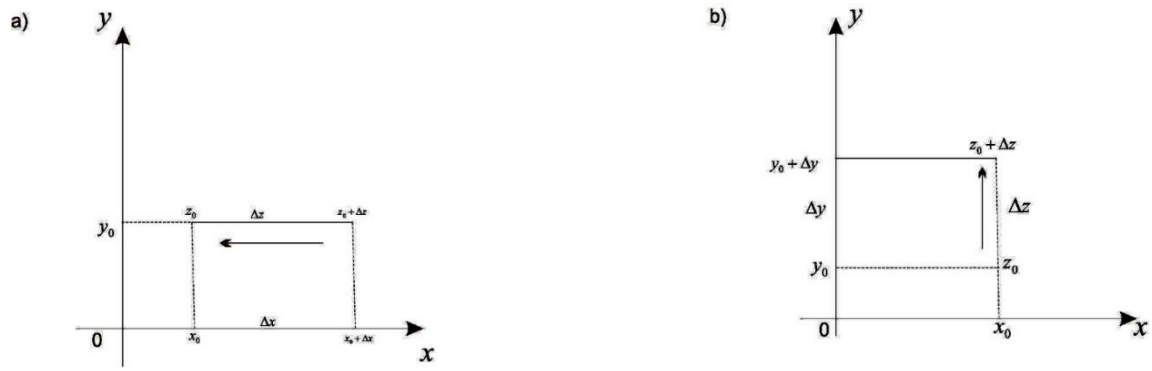
yoki $w = f(z) = U(x, y) + iv(x, y);\ \Delta w = \Delta u + i\Delta v$ bo`igani uchun (1.1)ni quyidagicha yozish mumkin;

$$f'(z) = \lim_{\Delta z \to 0} \frac{\Delta w}{\Delta z} = \lim_{\substack{\Delta x \to 0 \\ \Delta y \to 0}} \frac{\Delta u + i\Delta v}{\Delta x + i\Delta y} \quad (1.2)$$

Ta`rif. Agar $w = f(z)$ funksiya z_0 nuqtada hosilaga ega bo‘lsa, uni bu nuqtada differensiallanuvchi yoki monogen funksiya deyiladi.

Ta’rifdan ko‘rinadiki, agar $f(z)$ funksiya z_0 nuqtada hosilaga ega bo‘lsa, (1.1) limit mavjud bo‘lib, u $\Delta z = \Delta x + i\Delta y$ nolga qaysi yo‘l bilan intilishiga bog‘liq emas.

1-chizma



Demak, biz $z_0 + \Delta z$ nuqtani

z_0 nuqtaga $0x$ o‘qqa parallel yo‘l bilan intiltirishimiz mumkin. Bu holda $\Delta z = \Delta x$, $\Delta y = 0$ bo‘ladi (1a- chizma).

$$f'(z_0) = \lim_{\Delta x \to 0} \frac{\Delta u + i\Delta v}{\Delta x} = \lim_{\Delta x \to 0} \left(\frac{\Delta u}{\Delta x} + i\frac{\Delta v}{\Delta x}\right) = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x} \quad (1.3)$$

Xuddi shuningdek $z_0 + \Delta z$ nuqtani z_0 ga $0y$ ga parallel yo‘l bilan intiltirsak $\Delta x = 0$, $\Delta y = \Delta z$ bo‘ladi va (1.2) dan quyidagini hosil qilamiz (10b chizma):

$$f'(z_0) = \lim_{\Delta z \to_0} \frac{\Delta W}{\Delta z} = \lim_{\Delta y \to 0} \frac{\Delta u + i\Delta v}{i\Delta v} = \lim_{\Delta y \to 0} \left(\frac{\Delta v}{\Delta y} - i\frac{\Delta u}{\Delta y}\right) = \frac{\partial v}{\partial y} - i\frac{\partial u}{\partial y} \quad (1.4)$$

(1.3) va (1.4) lardan ushbu tengliklarni hosil qilish mumkin:

$$\frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x} = \frac{\partial v}{\partial y} - i\frac{\partial u}{\partial y} \Rightarrow \boxed{\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}}$$ (1.5) tengliklarga Koshi-Riman shartlari deyiladi.

Teorema. $w = f(z)$ funksiya z_0 nuqtada differensiallanuvchi bo‘lishi uchun

$u(x, y),\ v(x, y)$ funksiyalar z_0 nuqtada differensiallanuvchi va Koshi-Riman shartlarining bajarilishi zarur va yetarlidir.

Misol. $w = 2z - \bar{z}$ funksiya hosilaga ega ekanligi tekshirilsin.

Yechish. $w = 2x + 2iy - x + iy = x + i3y \Rightarrow u(x, y) = x, v(x, y) = 3y$

$\frac{\partial u}{\partial x}=1,\ \frac{\partial u}{\partial y}=0,\ \frac{\partial v}{\partial x}=0,\ \frac{\partial v}{\partial y}=3,$ bo‘lib, $\frac{\partial u}{\partial x}\neq\frac{\partial v}{\partial y}$ bo‘lgani uchun funksiya biror nuqtada ham hosilaga ega emas.

Misol. $W=3xy-3(x+y^2)i$ funksiyaning hosilasini toping

Yechish. $u(x,y)=3xy,\quad v(x,y)=-3(x+y^2),\ \frac{\partial u}{\partial x}=3y,\ \frac{\partial u}{\partial y}=3x,\ \frac{\partial v}{\partial x}=-3,\quad \frac{\partial v}{\partial y}=-6y,$ bo`lib, $x=1, y=0$.

Demak, funksiya (1;0) yoki $z_0=1+i0$ nuqtadagina hosilaga ega.

Kompleks o‘zgaruvchili funksiyalarni differensiallash qoidalari

Biz ko‘rdikki, $W=f(z)$ funksiyaning Z nuqtadagi hosilasi (differensiali)ni topish kerak bo‘lsa, quyidagi to‘rtta formulaning biridan foydalanish mumkin:

$f'(z)=\frac{\partial u}{\partial x}+i\frac{\partial v}{\partial x}$ (2.1), $f'(z)=\frac{\partial v}{\partial y}-i\frac{\partial u}{\partial y}$ (2.2), $f'(z)=\frac{\partial v}{\partial y}+i\frac{\partial v}{\partial y}$ (2.3),

$$f'(z)=\frac{\partial u}{\partial x}-i\frac{\partial u}{\partial y}(2{,}4)$$

Agar $W=f(z)$ funksiyaning haqiqiy va mavhum qismlari ajralmagan holda bo‘lsa, bu formulalardan foydalanish noqulay bo‘ladi. $f(z)$ funksiyaning hosilasini matematik analizdagi haqiqiy o‘zgaruvchining funksiyasi hosilasi qoidasi asosida topiladi, ya’ni:

1) $c'=0;$ 2) $z'=1;$ 3) $[f,(z)\pm f_2(z)]'=f_1'(z)\pm f_2'(z);$

4) $[f^n(z)]'=nf^{n-1}(z).f'(z)$

Misol $f(z)=e^{2z}$ funksiyaning hosilasini toping.

Yechish. $e^{2z}=e^{2x}(\cos 2y+i\sin 2y)\Rightarrow u(x,y)=e^{2x}\cos 2y,\ V(x,y)=e^{2x}\sin 2y.$

$\frac{\partial u}{\partial x}=e^{2x}\cos 2y,\ \frac{\partial u}{\partial y}=-2e^{2x}\sin 2y,\ \frac{\partial v}{\partial x}=2e^{2x}\sin 2y,\ \frac{\partial v}{\partial y}=2e^{2x}\cos 2y.$

Demak, $\frac{\partial u}{\partial x}=\frac{\partial v}{\partial y}$ va $\frac{\partial u}{\partial y}=-\frac{\partial v}{\partial x}$ Koshi-Riman sharti bajarilganligi uchun (2.1) formulaga ko‘ra: $f'(z)=2e^{2x}\cos 2y+i2e^{2x}\sin 2y=2e^{2x}(\cos 2y+\sin 2y)=2e^{2z}$

Javob: $f'(z)=2e^{2x}$.

Analitik funksiyalar

Ta’rif. Agar $w=f(z)$ funksiya E sohaning z_0 nuqtasida va uning atrofida ham differensiallanuvchi bo‘lsa, u shu nuqtada analitik deyiladi.

Ta’rif. Agar $w=f(z)$ funksiya E sohaning z_0 nuqtasida hosilaga ega bo‘lib, uning atrofida hosilaga ega bo‘lmasa, u holda funksiya z nuqtada monogen deyiladi.

Demak, funksiya biror nuqtada monogen bo‘lishidan, uning shu nuqtada analitik bo‘lishi kelib chiqmaydi.

Ta’rif. Agar $f(z)$ funksiya E sohaning barcha nuqtalarida hosilaga ega bo`lsa, u funksiya E da analitik deyiladi.

Ta’rif. $f(z)$ funksiya analitik bo‘lgan nuqtalar uning to‘g‘ri nuqtasi, analitik bo‘lmagan nuqtalar esa maxsus nuqtalar deyiladi.

Misol $w=x^2+y^2+ixy^3$ funksiyaning analitik yoki analitik emasligini tekshirilsin.

Yechish $u(x,y)=x^2+y^2,\quad v(x,y)=xy^3,\quad \frac{\partial u}{\partial y}=2x\quad \frac{\partial u}{\partial y}=2y,\ \frac{\partial v}{\partial y}=y^3,\quad \frac{\partial v}{\partial y}=3xy^2,$

$2x=3xy^2 \Rightarrow x(2-3y^2)=0 \Rightarrow x=0, y=\sqrt{\frac{2}{3}}$

$y^3=-2y \Rightarrow y(y^2+2)=0 \Rightarrow y=0$

Demak, faqat (0;0) dagina hosila mavjud, boshqa nuqtada hosila yo‘q, ya’ni funksiya analitik emas, $0(0;0)-$monogen nuqta.

Hosila argumentining va modulining geometrik ma’nosi

Biror E sohada $w=f(z)$ analitik funksiya berilgan bo‘lsin. Bu funksiya E dan olingan biror aniq z_0 nuqtada $f'(z_0)\neq 0$ hosilaga ega bo‘lsin. Bu funksiya yordami bilan (z) tekislikdagi z_0 nuqtani D dagi w_0 nuqtaga akslantirsak z_0 dan o‘tuvchi ixtiyoriy C_1, C_2 chiziqlar w_0 dan o‘tuvchi $\tilde{A}_1, \tilde{A}_2$ chiziqlarga akslanadi.

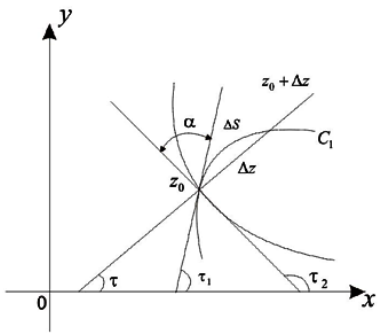
Biz $f'(z_0)\neq 0$ hosilaning argumenti va modulining geometrik ma’nosini ko‘raylik. Buning uchun $f'(z_0)$ kompleks sonni trigonometrik shaklga keltiraylik:

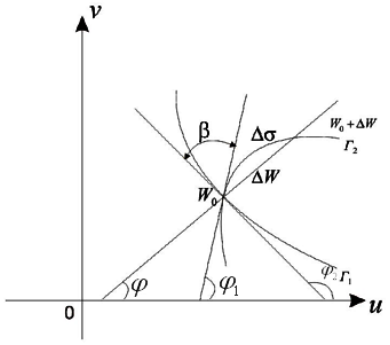
$f'(z_0)=R(\cos\phi+i\sin\phi),\quad R=|f'(z_0)|\neq 0-$cho`zilish koeffitsienti, $\phi=\arg[f'(z_0)]-$ burilish burchagi. Hosila ta’rifidan va 2-chizmadan foydalansak:

$$\phi=Arg\left[f'(z_0)\right]=\arg(\lim_{\Delta z\to 0}\frac{\Delta w}{\Delta z})=\lim_{\Delta z\to 0} arc\frac{\Delta w}{\Delta z}=\lim_{\Delta W\to 0}\arg\Delta w-\lim_{\Delta z\to 0}\arg\Delta z=\lim_{\Delta W\to 0}\varphi-\lim_{\Delta z\to 0}\tau=$$

$=\varphi_1-\tau_1 \Rightarrow \phi=\varphi_1-\tau_1$, ya’ni, C_1ning aksi Γ_1 bo‘lib u ϕ burchakka burilar ekan.

Xuddi shuningdek C_2 ning aksi Γ_2 hosil bo‘lib, u ham ϕ burchakka burilishini ko‘rsatish





mumkin ya’ni $\phi=\varphi_2-\tau_2$ (4.2)

2-chizma

(4.1) va (4.2) larni tenglab ushbuni $\varphi_1-\tau_1=\varphi_2-\tau_2$ hosil qilamiz yoki $\varphi_2-\varphi_1=\tau_2-\tau_1$ bundan $\alpha=\beta$ (4.3) ekanligi kelib chiqadi.

Shunday qilib, $w=f(z)$ analitik funksiya yordami bilan E sohani D sohaga akslantirsak z_0 nuqtadan o‘tuvchi C_1, $C_2\ldots$ chiziqlarning hammasi D da bir xil $\hat{o}=\arg f'(z_0)$ burchakka burilar ekan. C_1va C_2 chiziqlar orasidagi burchaklar o‘zgarmay akslanadi ya’ni $\alpha=\beta$ bo‘ladi.

Xossa. Analitik funksiya yordami bilan bajariladigan akslantirish hosila nolga teng bo‘lmagan barcha nuqtalarda burchaklarni saqlash xossasiga ega.

$R=|f'(z_0)|=\left|\lim_{\Delta z\to 0}\frac{\Delta W}{\Delta z}\right|=\lim_{\Delta z}\left|\frac{\Delta W}{\Delta z}\right|=\lim_{\Delta z\to 0}\frac{\Delta\sigma}{\Delta s}$ (4.4) ga z_0 nuqtadan o‘tuvchi har qanday

$C-$chiziqning cho‘zilish koeffitsienti deyiladi. Boshqacha aytganda $W=f(z)$ analitik funksiya yordamida bajariladigan akslantirish jarayonida istalgan kichik yoy z_0 nuqtaning kichik atrofida $R=|f'(z_0)|$ marta o‘zgaradi. z_0 dan o‘tuvchi istalgan barcha chiziqlar uchun cho‘zilish koeffitsienti bir xil bo‘ladi, chunki $z_0+\Delta z$ ning z_0 ga intilishi ixtiyoriydir.

Xossa. Analitik funksiya yordami bilan bajariladigan akslantirish $f'(z)$ hosila nolga teng bo‘lmagan barcha nuqtalar o‘zgarmas $R=|f'(z_0)|$ cho‘zilishga ega

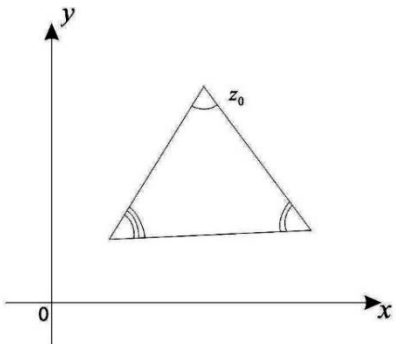
I va II tur konform akslantirishlar

Ta’rif. Agar $W=f(z)$ analitik funksiya yordamidagi akslantirish natijasida z_0 nuqtadagi cho‘zilish koeffitsienti va burchakning kattaligi ham o‘zgarmasa u I-tur konform akslantirish deyiladi.

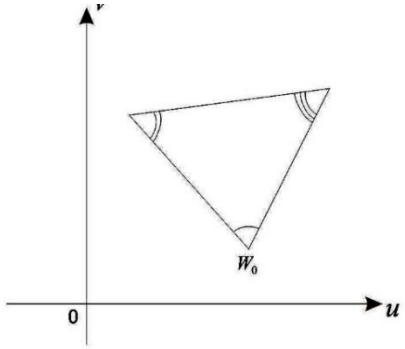
Izoh, Konform – conformis lotincha so‘z bo‘lib, o‘xshash demakdir.

Misol ushun bir uchi z_0 nuqtadan iborat istalgancha kichik uchburchakni $W=f(z)$ analitik funksiya yordami bilan akslantirsak, bir uchi W_0 nuqtadan iborat egri chiziqli uchburchakka ega bo‘lamiz. Lekin ikkala uchburchakdagi mos burchaklar o‘zaro teng bo‘ladi (3-chizma).

Endi $x0y$ va $u0v$ koordinata tekisliklarini bitta tekislikka joylashtiraylik, $W=\overline{z}=x-iy$ funksiya vositasi bilan $z=x+iy$ nuqtalarni akslantirsak, o‘sha tekislikda $0x$ (demak, $0y$) o‘qqa nisbatan simmetrik nuqtalar paydo bo‘ladi.

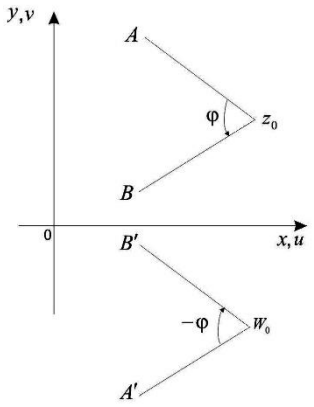


3-chizma



4-chizma

Shunga asosan, z_0 nuqtadan chiqqan ikki to‘g‘ri chiziq va ular orasidagi φ burchakni akslantirsak, xuddi ko‘zguda tushirilgandek $0x$ o‘qqa nisbatan simmetrik bo‘lib tushadi. Shuning uchun φ burchakning aksI ($-\varphi$), ya’ni qarama-qarshi tomonga yo‘nalgan bo‘lib tushadi.



Ta’rif. Agar funksiya yordami bilan akslantirish natijasida z nuqtadagi cho‘zilish o‘zgarmasa va burilish burchagining hamma kattaligi (miqdori) o‘zgarmay, faqat yo‘nalishi qarama-qarshisiga o‘zgarsa, u II-tur konform akslantirish deyiladi.

Shunday qilib, $w=f(z)$ analitik funksiyaga qo‘shma bo‘lgan $\overline{w}=\overline{f}(z)$ funksiya yordamida amalga oshiriladigan akslantirish II tur konform akslantirish bo‘lar ekan.

Misol $w=z^2$ funksiya yordamida bajariladigan akslantirishda $z_0=\frac{1}{2}$ nuqtadagi cho‘zilish va burilish burchagini toping.

Yechish. $w' = 2z$ bo`lib $f'(\frac{1}{2}) = 2 \cdot \frac{1}{2} = 1 \neq 0$; $1 = \cos 0 + i \sin 0$; $R = |f'(z_0)| = 1$, ya'ni cho‘zilish koeffitsienti 1 ga teng bo‘lib, cho‘zilish ro‘y bermaydi.

$\Phi = \arg z'(\frac{1}{2}) = 0$, ya'ni burilish burchagi 0 ga teng bo‘lib, burilmaydi.

ASOSIY ADABIYOTLAR

1. O‘.Toshmetov, R.M.Turg‘unboyev, E.M.Saydamatov, M.Madirimov “Matematik analiz” (1-qism), (179-182b).Toshkent. 2015.
2. Sh.A.Ayupov, M.A.Berdiqulov, R.M.Turg‘unboyev “Funksional analiz” .T.2007.
3. S.X. Sirojiddinov, M.Maqsudov, M.S.Salohiddinov. Kompeleks o`zgaruvchining funksiyalari nazaryasi-T,: O`qtuvchi, 1979
4. G.G‘ayimnazarov, O.G.G‘ayimnazarov Funksional analiz kursidan masalalar echish.T:-“Fan va texnalogiya” 2006.
5. Alimov A.A., Berdiqulov M.A. “Reshenie zadach po funksionalnomu analizu”. T.2005.
6. Ayupov Sh.A., Berdiqulov M.A., Turg‘unboyev R.M. “Funksiyalar nazariyasi” .T.2004y.-146b.
7. Bakhridinovich A. B. Theoretical bases of formation of design-design competence of future engineers in the process of higher education. – 2022.
8. Adilov B. ORGANIZATIONAL AND PEDAGOGICAL FOUNDATIONS OF THE FORMATION OF PROJECT-DESIGN COMPETENCE OF STUDENTS OF THE ENGINEERING DIRECTION //Science and innovation. – 2022. – T. 1. – №. B4. – C. 318-322.